Another Hanukkah Miracle:
The Gaps Between Consecutive Christmas-in-Hanukkah Years is ALWAYS a Fibonacci Number!

By Shalosh B. EKHAD

In fond memory of Marvin Isadore Knopp (1933-2011) z"l, who left us this Christmas Eve, the fifth candle of Hanukkah 5772, who did **not** *like computers (even for Email), but was nevertheless one of the kindest and warmest* **neshamot** *(souls) on this planet.*

Jane Legrange, the beloved wife of my beloved master, commented this last Christmas Eve that it is so nice to be able to light a *menorah* (or *chanukiah*) at the same time as the (Christian) *goyim* sitting next to their Christmas trees, and she asked her husband, who asked me, how special is that coincidence? Using the Maple package

`http://www.math.rutgers.edu/~zeilberg/tokhniot/LUACH`

written by Zeilberger a few years ago, and updated and upgraded recently in order to answer this question, I figured out that in this, third, millenium, it happens 270 times, so %27 of the time. In the next two millenia after that it would happen 266 times in each. In the sixth millenium (i.e. between 5001 and 6000), it would only happen 263 times, while in the seventh millenium (i.e. between 6001 and 7000), it would only happen 258 times. If we are still alive in the eighth millenium (i.e. between 7001 and 8000), it would only happen 134 times, and in the ninth millenium, only 15 times! After that it will never happen for many many years, certainly not until 20000 AD. The last year is 8473, where we would light the first candle on Christmas Eve. After that Hanukkah will always be way **after Christmas**.

But cheer up! Eventually we can, every few years, sit in a Sukkah during Christmas Eve! The first time this would happen is 16103, when Christmas would be on the 20th day of Tishrei, and we can put the Christmas tree inside the Sukkah. During the 17th millenium (between 16001 and 17000) we would only have 68 Sukkah Christmases, but during the 18th (between 17001 and 18000) we would have 207 of them, during the 19th we would have 239 but only 235 during the 20th millenium.

While doing these calculations, I noticed something really amazing! Between the Gregorian years of 1801 and 7390 (when Christmas-in-Hanukkha is starting to get rarer and rarer until it disappears completely) the gap between consecutive such lucky years is **always** a Fibonacci number, in fact, it is always a member of the set $\{2, 3, 5, 8\}$. It would be interesting to have a *conceptual* proof, in addition to my computational proof.

And guess what! Once Christmas-in-Sukkot becomes not too rare, and that would happen in 17064 A.D., we have the same phenomenon, the gaps between consecutive Christmas-in-Sukkot years is also **always** a member of the set $\{2, 3, 5, 8\}$, at any rate, until 20000 A.D. Of course, eventually Christmas will say good-bye to Sukkot also, and start visiting Rosh Hashana (but not that often since the latter is only two-days long), then Shavuot, and then Pesach. Readers are welcome to experiment with the Maple package `LUACH` to continue these preliminary investigations.

For a list of all the Christmas-in-Hanukkah years (between 1801 and 20000 (of course it ends in 8478)) see,

`http://www.math.rutgers.edu/~zeilberg/tokhniot/oLUACHs1`

and for a list of all the Christmas-in-Sukkot years (also between 1801 (of course it only starts in 16103) and 20000), see

`http://www.math.rutgers.edu/~zeilberg/tokhniot/oLUACHs2` .

Shalosh B. Ekhad, c/o D. Zeilberger, Department of Mathematics, Rutgers University (New Brunswick), Hill Center-Busch Campus, 110 Frelinghuysen Rd., Piscataway, NJ 08854-8019, USA.

Email: `c/o zeilberg at math dot rutgers dot edu` ,

Written: Jan. 2, 2012 .